Full-Wave Analysis of a Transversely Magnetized Ferrite Nonradiative Dielectric Waveguide

Amílcar Careli César and Rui Fragassi Souza

***Abstract*— A full-wave analysis is applied to a nonradiative dielectric waveguide where the isotropic dielectric slab is replaced by a transversely magnetized ferrite. The characteristic equation is obtained and the corresponding effects are discussed. The above structure exhibits reciprocal propagation characteristics. Nonreciprocal effects are also possible with a proper dielectric loading. Several numerical results are presented in the form of dispersion curves and operational diagram, as function of several ferrite and guide parameters. Electronically tuned and nonreciprocal devices can be implemented using this simple structure.**

I. Introduction

Nonradiative dielectric (NRD) waveguide was proposed by Yoneyama and Nishida [1] for millimeter-wave applications. The basic structure consists of an isotropic dielectric slab placed between two parallel conducting planes separated by a distance less than half a wavelength. The propagating modes are hybrids and usually referred as LSM or LSE modes. The mode of interest is the LSM_{01} mode, which has an electric field configuration that is predominantly parallel to the conducting planes.

Several properties of the NRD waveguide have been investigated, like coupling between slabs, discontinuities, curvature effects, etc. [2]–[4]. Due to its simple construction, devices like filters, junctions, transitions and radiating structures have been implemented using this technique [5]–[7]. Some of these devices were combined to build a transmitter and a receiver front end at 35 GHz [8].

Its superior characteristics, such as low losses and suppression of undesired radiation, have encouraged the proposal of some modified NRD structures [9], [10]. Only recently have the analysis and construction of nonreciprocal NRD structures been dealt with [11].

This work investigates the propagation in an NRD structure where the isotropic dielectric slab is replaced by a transversely magnetized ferrite. Dispersion curves, operational diagram, and some other numerical results are presented.

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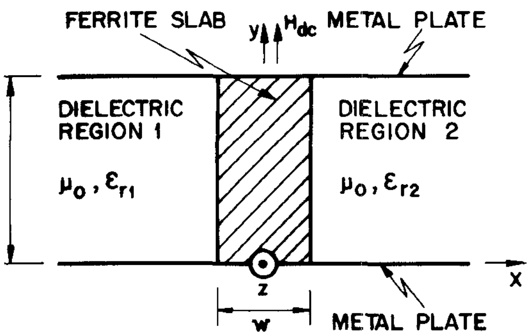

Fig. 1. Geometry of an NRD waveguide with a ferrite slab.

II. Theory

The general structure analyzed is shown in Fig. 1. The surrounding media consist of two isotropic dielectrics with relative dielectric constants equal to ε_{r1} and ε_{r2}. The ferrite slab (dimensions h, W) is magnetized in the y-direction and is assumed lossless, saturated and subjected to small signals. The electromagnetic fields are supposed proportional to exp $(-j\beta z)$ where β is the propagation constant, and to exp $(j\omega t)$ where ω is the signal radian frequency. Outside of the ferrite slab the fields are assumed to be evanescent.

Since the applied dc field is parallel to the y-axis, the ferrite (dielectric constant ε_r, Landé factor g, saturation magnetization $4\pi M_s$) is described by the permeability tensor [12]:

$$\bar{\bar{\mu}} = \mu_0 \begin{bmatrix} \mu_1 & 0 & j\delta \\ 0 & 1 & 0 \\ -j\delta & 0 & \mu_1 \end{bmatrix} \tag{1}$$

where the elements μ_1 and δ are determined in the usual way [13]. Helszajn [14] suggests that, for this given dc applied field and propagation direction, the components of the RF electric and magnetic fields inside the ferrite slab ($|x| \le W/2$) can be found by using a similar procedure to that employed in the case of a circular waveguide [15]. Therefore, we find the following x, y and z field components inside the ferrite slab, shown in (2a)–(2f) at the bottom of the next page, where $\eta_0 = 377\,\Omega$, $k_0 = 2\pi/\lambda_0$, $k_y = n\pi/h\,(n = 1, 2, 3 \ldots)$, and $q_{1,2} = [(s_{1,2} - a)\,s_{1,2}]/(bk_y)$. $s_{1,2}$ are the roots of the quadratic equation $s^2 - (a + c)s + (ac - bd) = 0$, where $a = k_1^2(\delta/\mu_1) - k_\delta^2$; $b = -\eta_0 k_0(\delta/\mu_1)$; $c = k_1^2/\mu_1$; $d = -(\delta\varepsilon_r k_0/\eta_0\mu_1)\,k_y^2$; $k_1^2 = k^2 - k_y^2$; $k_\delta^2 = \delta\varepsilon_r k_0^2$ and $k^2 = \mu_1\varepsilon_r k_0^2$. The functions u_1 and u_2 are selected in accordance with the geometry of the structure. The appropriate choice that satisfies the boundary conditions at the conducting plane interfaces are

$$u_1 = (A\cos\alpha_1 x + B\sin\alpha_1 x)\cos k_y y \tag{3a}$$

$$u_2 = (C\cos\alpha_2 x + D\sin\alpha_2 x)\cos k_y y \tag{3b}$$

where A, B, C and D are coefficients to be determined and $\alpha_{1,2}^2 = s_{1,2} - \beta^2$.

Outside the ferrite slab ($|x| \geq W/2$) the fields decay according to exp $[-p_{1,2}(|x| - W/2)]$, where the subscripts 1, 2 refer to the dielectric regions defined by $x \leq -W/2$ and $x \geq W/2$, respectively. The y-direction components of the RF electric (E_y^d) and magnetic (H_y^d) fields in these dielectric media must satisfy the wave equations

$$(\nabla_t^2 + \gamma_{1,2}^2)E_y^d,\ H_y^d = 0 \tag{4}$$

where $\gamma_{1,2}^2 = \varepsilon_{r1,2}k_0^2 - k_y^2$ and ∇_t is the transverse component of the nabla operator. In this case $p_{1,2}^2 = \beta^2 + k_y^2 - \varepsilon_{r1,2}k_0^2$.

The application of the boundary conditions at the dielectric-ferrite interfaces results in a system of homogeneous equations for the coefficients A, B, C, and D, that can be written in the matrix form

$$[K]\,[A\,B\,C\,D]^T = 0 \tag{5}$$

where $[K]$ is a 4×4 matrix and the superscript T denotes the transpose matrix. For a non-trivial solution, we must have

$$\det[K] = 0 \tag{6}$$

The characteristic equation for all the hybrid propagating modes in the ferrite NRD structure is found by developing (6). Numerical solutions of (6) were obtained, furnishing dispersion relations for several modes. The propagating modes have all the six field components and are classified as $E_{\ell m,n}^x$ and $H_{\ell m,n}^x$, where "x" denotes TM and TE x-direction modes, respectively. The subscript "ℓ" is used to designate a hybrid mode which corresponds, in the limit condition of $\mu \rightarrow 1$ and $\delta \rightarrow 0$, to the LSM or LSE mode in a conventional NRD guide. A similar nomenclature was used to classify the modes of a circular waveguide containing a ferrite rod [16]. The integer "n " is the nth value for k_y and " m" (equal to 1, 2, 3, ...) is the mth root of (6). In particular, the lowest order $E_{\ell m,n}^x$ mode is the $E_{\ell 11}^x$ mode and the lowest order $H_{\ell m,n}^x$ mode is the $H_{\ell 11}^x$ mode.

The Nonreciprocal Effect

A close analysis of the characteristic equation (equation (6)) reveals that if $\varepsilon_{r1} = \varepsilon_{r2}$ the terms involving odd powers of the propagation constant cancel themselves and the structure is reciprocal. A simple way to obtain a nonreciprocal effect is to provide a proper dielectric loading. Therefore, if $\varepsilon_{r1} \neq \varepsilon_{r2}$ the characteristic equation exhibits numerically different solutions for the forward and reverse propagation directions.

III. Numerical Results

A program in the Pascal language was written to run in a microcomputer for solving equation (6). A numerical check of the program with some results of other author [17] was made. In this sense, the normalized propagation constant for the $E_{\ell 11}^x$ and $H_{\ell 11}^x$ modes of a ferrite NRD guide in the isotropic limit condition ($\mu_1 = 1.0$ and $\delta = 10^{-6}$) were compared to the conventional NRD guide LSM_{01} [17] and LSE_{10} for normalized thickness varying from near cut-off condition until $\beta/\sqrt{\mu_1\varepsilon_r}.k_0 \cong 0.95$. The differences observed were within 10^{-6}. Similar agreement was obtained in the field profiles.

Numerical results are shown in Fig. 2 through Fig. 6 for a commercially available ferrite slab operating at 50 GHz with the following parameters: $4\pi M_s = 4.0$ kG, $g = 2.22$ and $\varepsilon_r = 12.3$. Fig. 2 shows normalized dispersion curves for the first few hybrid modes in a ferrite NRD guide operating at 50 GHz and under a static magnetic field of 1.5 kOe. The desired mode is the $E_{\ell 11}^x$ that approaches the LSM_{01}

$$E_y^f = s_1u_1 + s_2u_2 \tag{2a}$$

$$H_y^f = -\frac{1}{k_y}\left(q_1\frac{\partial u_1}{\partial y} + q_2\frac{\partial u_2}{\partial y}\right) \tag{2b}$$

$$E_x^f = \frac{\partial^2}{\partial x\partial y}(u_1+u_2) + \frac{\mu_1\beta}{\delta k_y^2}\left[(s_1-a)\frac{\partial u_1}{\partial y} + (s_2-a)\frac{\partial u_2}{\partial y}\right] \tag{2c}$$

$$H_x^f = \frac{1}{\eta_0\delta k_0}\left[(k_1^2-s_1)\frac{\partial u_1}{\partial x} + (k_1^2-s_2)\frac{\partial u_2}{\partial x}\right] - \frac{\varepsilon_r k_0\beta}{\eta_0}(u_1+u_2) \tag{2d}$$

$$E_z^f = -j\left\{\beta\frac{\partial}{\partial y}(u_1+u_2) + \frac{\mu_1}{\delta k_y^2}\left[(s_1-a)\frac{\partial^2 u_1}{\partial x\partial y} + (s_2-a)\frac{\partial^2 u_2}{\partial x\partial y}\right]\right\} \tag{2e}$$

$$H_z^f = -j\left\{\frac{\beta}{\eta_0\delta k_0}\left[(k_1^2-s_1)u_1 + (k_1^2-s_2)u_2\right] - \frac{\varepsilon_r k_0}{\eta_0}\frac{\partial}{\partial x}(u_1+u_2)\right\} \tag{2f}$$

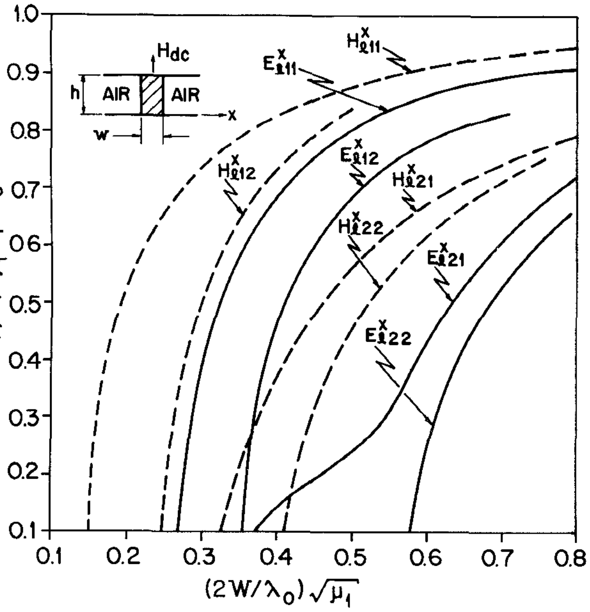

Fig. 2. Normalized propagation constant variation with normalized thickness for a few hybrid modes in a reciprocal case. The desired mode is the $E^x_{\ell 11}$ (approaches the LSM_{01} mode when $\mu_1 \rightarrow 1$ and $\delta \rightarrow 0$). Guide and operation parameters are $(W/h)\sqrt{\mu_1} = 0.35$ and $H_{dc} = 1.5$ kOe.

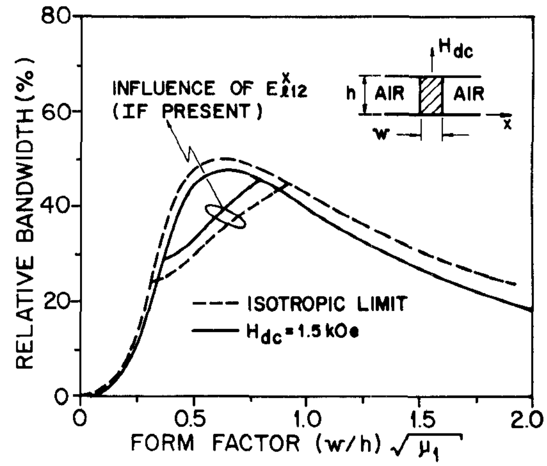

Fig. 4. Relative bandwidth in the monomode operation for the ferrite NRD waveguide. The corresponding isotropic limit ($\varepsilon_r = 12.3$, $\mu_1 = 1.0$ and $\delta = 10^{-6}$) is also shown for comparison.

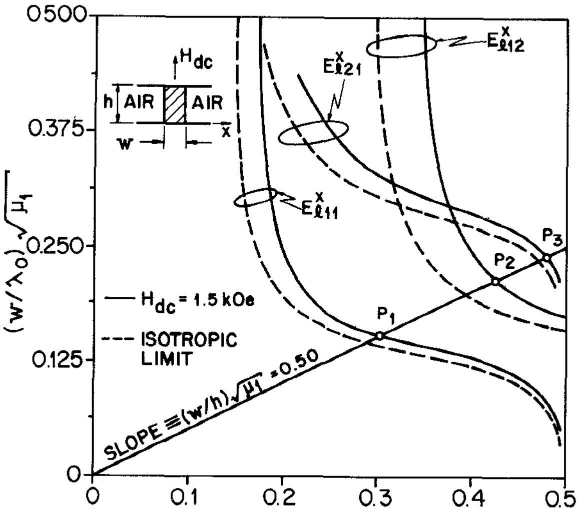

Fig. 3. Operational diagram of an NRD guide with a ferrite slab. The straight line passing through points P_1, P_2 and P_3 is useful to obtain the bandwidth in the monomode operation.

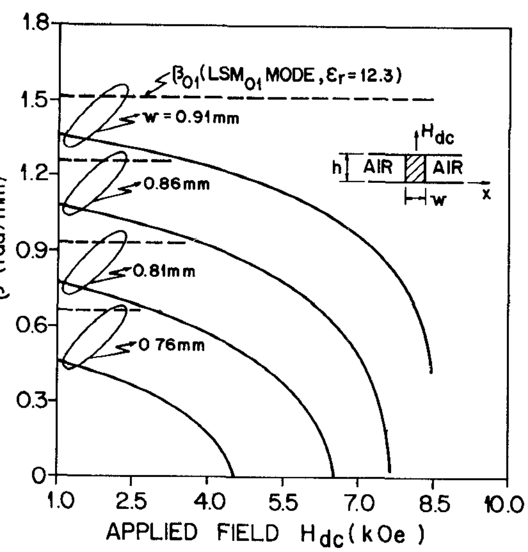

Fig. 5. Influence of the biasing magnetic field on the $E^x_{\ell 11}$ mode propagation constant for $h = 2.57$ mm and several values of slab widths. Dashed lines are the corresponding propagation constants for conventional NRD ($\varepsilon_r = 12.3$) LSM_{01} mode.

mode in the isotropic limit. Solution of the characteristic equation reveals that the lowest cut-off frequency hybrid mode is the $H^x_{\ell 11}$ mode, that approaches the LSE_{10} in the isotropic limit. The ferrite NRD dimensions must satisfy monomode and nonradiative operations and the corresponding propagation constant can be obtained from diagrams such as Fig. 2. For instance, using metal-plate separation $h = 2.57$ mm and ferrite slab width $W = 0.91$ mm, the propagation constant for the $E^x_{\ell 11}$ mode is $\beta = 1.35$ rad/mm.

The operational diagram of a ferrite NRD guide is shown in Fig. 3. The mode diagram changes but the nonradiative characteristic of the structure is maintained by going from zero to a finite applied magnetic field. Points P_1, P_2 and P_3 are useful to establish the bandwidth for monomode operation (see, for instance, the straight line of slope 0.50 in Fig. 3). It can be observed the minor influence, in that bandwidth, caused by the presence of the $E^x_{\ell 12}$ mode when compared with the isotropic operation condition. This is best observed in Fig. 4, where the relative bandwidth is defined in accordance with [17]. The relative bandwidth is smaller than that corresponding to the isotropic condition, but is still sufficient large for practical purposes.

Fig. 5 shows the influence of the biasing magnetic field on the propagation constant for the $E^x_{\ell 11}$ mode of a ferrite NRD guide. The propagation constant for the corresponding isotropic limit LSM_{01} mode is also presented to show the difference. An increase in the dc applied field (H_{dc}) will increase this difference. For a given geometry the $E^x_{\ell 11}$ mode tends to the cut-off condition for high values of H_{dc}, resulting in the sharp drop in β curves.

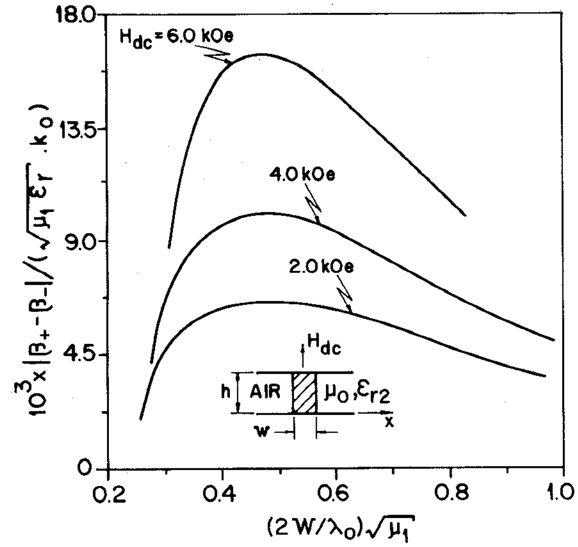

Fig. 6. Normalized differential propagation constants for the $E^x_{\ell 11}$ mode for several values of biasing magnetic field. This nonreciprocal effect was obtained for $\varepsilon_{r1} = 1.0$ (air), $\varepsilon_{r2} = 6.8$ and $(W/h)\sqrt{\mu_1} = 0.5$.

The possible nonreciprocal effect associated with the $E^x_{\ell 11}$ mode is presented in Fig. 6, in a particular case. β_+ corresponds to the solution of the characteristic equation for the forward propagation and β_- to the reverse propagation. Increasing values of nonreciprocal effect are obtained with stronger values of H_{dc}. The guide must operate with a differential propagation constant lower than the maximum value to guarantee nonradiative and monomode operations. The maximum value occurs above the cut-off condition of the nearest higher order mode. It was also observed that the nonreciprocal effect increases with increasing values of ε_{r2} for a given value of ε_{r1}. Alternatively, strong nonreciprocal effects in the millimeter-wave region can be obtained by other structures [11], [18]. The dielectric semi-space with relative permittivity ε_{r2} must only have sufficient width to fit the evanescent field profile.

IV. Conclusions

This work presented a theoretical investigation of the wave propagation in an NRD structure with a transversely magnetized ferrite using a full-wave approach. The procedure to obtain all the electric and magnetic field components and the characteristic equation was shown. Several numerical results are presented as functions of geometrical dimensions, ferrite parameters and operating conditions. For the limiting case where the ferrite approaches an isotropic dielectric, the desired $E^x_{\ell 11}$ mode approaches the LSM_{01} mode. These informations allow the identification of the fields distribution and the consequent implementation of NRD electronically tuned and nonreciprocal devices.

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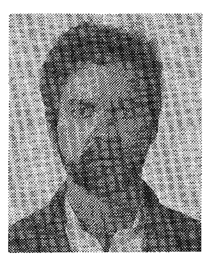

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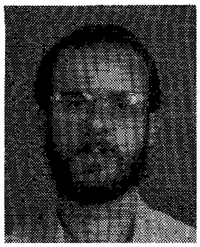

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